Effective-susceptibility tensor for a composite with ferromagnetic inclusions: Enhancement of effective-media theory and alternative ferromagnetic approach

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For calculating magnetic properties of a composite usually effective-medium theories are used. However, we show that for a composite with ferromagnetic inclusions such theories, in particular, Maxwell-Garnett equation, give peculiar and unphysical results, such as significant shift of ferromagnetic-resonance frequency with diminishing volume fraction of ferromagnetic inclusions. Starting from ferromagnetic theory we derive a simple expression for the calculation of the effective magnetic susceptibility of a composite and follow with detailed magnetostatic derivation of tensor equivalent of Maxwell-Garnett equation. By demonstrating the equivalence of both derivations we confirm the validity of the expression which we obtained from the ferromagnetic theory. Furthermore, we identify errors leading to unphysical results of effective-medium theories and show the correct application of these theories.
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I. INTRODUCTION

Determining the mixture properties as a function of constituents' properties has found use in a range of fields, particularly in the field of electric properties. There are several expressions for evaluating the effective permittivity of composite (here taken as mixture of matrix and inclusions),[1–4] most notably Bruggemann effective-medium theory (EMT) for denser composites and Maxwell-Garnett (MG) equation for dilute (noninteracting) composites with spherical inclusions (particles).[1,2] In the limit of low volume fraction all expressions reduce to the MG equation [1,2]

$$\frac{\varepsilon_{\mathrm{eff}}-\varepsilon_m}{\varepsilon_{\mathrm{eff}}+2\varepsilon_m}=F\frac{\varepsilon_p-\varepsilon_m}{\varepsilon_p+2\varepsilon_m}, \tag{1}$$

where F is the volume fraction of particles (inclusions) and subscript p refer to particle (inclusion), m to matrix, and eff to effective property.

On the basis of equivalence of electric- and magnetic-field equations in the absence of the electric charge and current sources the same equations can be used also for calculation of the effective magnetic permeability by simply exchanging permittivity ε with permeability μ.[2,3] Such effective-medium expressions for the effective magnetic permeability are widely used in literature.[3–14] Yet, as we show later they have only little physical relevance in case of composites with ferromagnetic inclusions (particles), especially when permeability (susceptibility) as a function of frequency is evaluated in the range of the ferromagnetic resonance.

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The aim of this paper is to determine an unambiguous expression for the effective permeability of a dilute composite with ferromagnetic inclusions, valid in whole frequency range and correctly reproducing the tensor nature of the ferromagnetic susceptibility. In order to achieve this we first qualitatively compare the magnetic-susceptibility frequency spectrum of a dilute composite, calculated from the ferromagnetic theory and the Maxwell-Garnett equation. Further, we quantitatively derive our expression for an effective-susceptibility tensor from the ferromagnetic theory and continue with a derivation of the magnetostatic equivalent of Maxwell-Garnett equation for a susceptibility tensor. We show that although the Maxwell-Garnett equation as used[3,5–14] is not valid even far from resonance, ferromagnetic theory and magnetostatics give physically correct expressions, however, the magnetostatic expressions are significantly more complex. Finally, by comparing both expressions we conclude that our ferromagnetic expression is more elegant for calculation of an effective susceptibility of dilute and homogeneous composite with ferromagnetic inclusions.

II. FERROMAGNETIC RESONANCE IN COMPOSITE

In the study of ferromagnetic materials one is interested not only in the magnetic susceptibility but often even more in its frequency dependence. The frequency dependence of the magnetic susceptibility of a homogeneous ferromagnetic material is obtained from the theory of ferromagnetism (e.g., Ref. 15), with notable exception of a superparamagnetic single-domain particle, which pose somewhat different and more complex problem (e.g., Ref. 16) and shall not be considered here.

The ferromagnetic susceptibility due to the magnetization rotation is a tensor

$$\boldsymbol{\chi}=\begin{vmatrix}\chi & -\kappa \\ \kappa & \chi\end{vmatrix}, \tag{2}$$

with the following components:[15]

$$\chi=\frac{\omega_M[\omega_o+i\alpha\omega+(N_{x,y}-N_z)\omega_M]}{\omega_r^2-\omega^2+2i\alpha\omega[\omega_o+(N_x+N_y-2N_z)\omega_M/2]},$$
$$\kappa=\frac{i\omega\omega_M}{\omega_r^2-\omega^2+2i\alpha\omega[\omega_o+(N_x+N_y-2N_z)\omega_M/2]}, \tag{3}$$

where α is a dimensionless damping coefficient and $\omega_o=\gamma H_o$, $\omega_M=\gamma 4\pi M_s$ with H_o denoting static magnetic field, M_s the saturation magnetization, and γ the gyromagnetic constant. The resonance frequency ω_r is defined by Kittel's equation as

$$\omega_r^2=[\omega_o+(N_x-N_z)\omega_M][\omega_o+(N_y-N_z)\omega_M] \tag{4}$$

and is essentially determined by the static magnetic field H_o. Equation (3) gives the general form of the susceptibility components, where the difference between bulk and finite sample is set with demagnetization factors. Demagnetization factors N_i are a function of shape (e.g., $N_{x,y,z}=1/3$ for sphere), but for bulk material N_i are not defined and are therefore omitted in the expressions.[15] It is evident that the susceptibility of the bulk material and that of the spherical particle are equivalent. Also the resonance frequency ω_r is equivalent in both cases.

For the demagnetized or partially magnetized particles the susceptibility is also derived from the theory of ferromagnetism,[17–19] but the expressions greatly depend on the geometry and the domain configuration of the particles. Nevertheless, there are some simple expressions for the ferromagnetic permeability tensor for a few special cases of demagnetized particles.[17,18] These permeability tensors are diagonal and, importantly, are going through the resonance at the same frequency as the magnetized particle.

Let us now consider the composite made from well-dispersed spherical ferromagnetic particles surrounded by nonmagnetic matrix and so dilute that the interparticle interactions are negligible. Intuitively, one would expect that in an assembly of identical ferromagnetic particles without interparticle interactions every particle is subjected to the same magnetic field, being equal to the external magnetic field. Hence, also the resonance of the magnetization in particles occurs at the same magnetic-field frequency (resonance frequency of a single particle) and to distant observer the magnetization of the whole composite sample would have the resonance at the resonance frequency of an individual particle, given by Kittel's equation (4). This argument applies for all types of ferromagnetic particles that can be used in composites, namely, demagnetized, partially magnetized, or magnetized (magnetized with external static magnetic field or single-domain particles).

Alternatively, the frequency dependence of the permeability in such composite is frequently calculated with the effective-medium equations (magnetostatic case).[3–14] The MG equation [Eq. (1)] in case of nonmagnetic matrix ($\mu_m=1$) has the following form for the susceptibility ($\chi=\mu-1$):

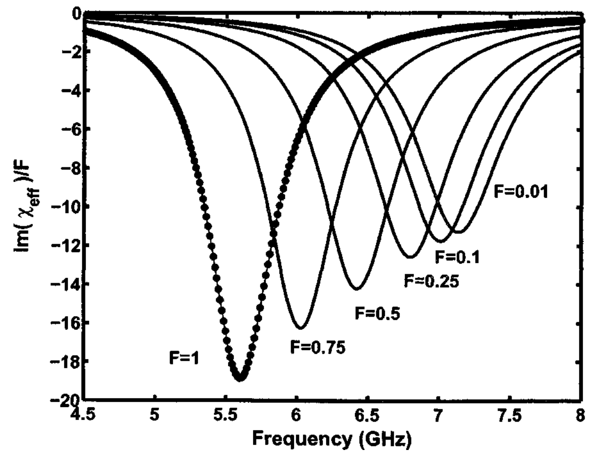

FIG. 1. Normalized imaginary part of effective susceptibility ($\chi''_{\rm eff}$) as a function of frequency, calculated with two expressions for various volume fractions of ferromagnetic particles: Maxwell-Garnett equation [Eq. (5)] (solid lines), and Eq. (13), that we derived (symbol •). In the latter case the curves are identical for all volume fractions and equal to Maxwell-Garnett curve for $F=1$. As an intrinsic susceptibility the diagonal susceptibility χ from Eq. (3) was taken, using zero demagnetization factors and parameters $\omega_o=35.2\times10^9\ \mathrm{s}^{-1}$, $\alpha=0.05$, and $\omega_M=66.3\times10^9\ \mathrm{s}^{-1}$.

$$\chi_{\rm eff}=F\frac{3\chi_p}{3+\chi_p(1-F)}. \tag{5}$$

Since the spherical shape of the particles was already accounted for in derivation of MG equation, one have to insert the diagonal component of a ferromagnetic susceptibility for bulk material [Eq. (3) with $N_i=0$] as the intrinsic permeability in the Maxwell-Garnett equation [Eq. (5)]. This gives the effective susceptibility of the composite as a function of frequency, as shown in Fig. 1. The effective susceptibility was calculated with Eq. (5) for different volume fractions of magnetized ferromagnetic particles in composite and divided by volume fraction for normalization.

For volume fractions $F\to1$ the resonance frequency matches that of the bulk material. With decreasing volume fraction the resonance frequency of the composite increases and approaches the following limit:

$$\omega_r^2|_{F\to0}=\omega_o(\omega_o+\omega_M/3). \tag{6}$$

Here we must recall previous statement that in dilute composite with vanishing interparticle interactions the resonance frequency of individual spherical particles should be given by Kittel's equation. In ferromagnetic theory both bulk resonance frequency ($F\to1$) and that of the free spherical particle ($F\to0$) are equal and any resonance-frequency shift could occur only due to the interparticle interactions in the intermediate region of F. So, instead of resonance frequency approaching the resonance frequency of an individual particle ($\omega_r=\omega_o$) the Maxwell-Garnett equation (5) gives a different resonance-frequency limit [Eq. (6)] having no physical meaning.

To resolve this apparent discrepancy we analyzed in detail both ferromagnetic and magnetostatic approach for calculation of the effective susceptibility.

III. FERROMAGNETIC DERIVATION OF THE EFFECTIVE SUSCEPTIBILITY

The dynamics of the magnetization vector $\mathbf{M}$ in the magnetized ferromagnetic material is based on the well-known equation of motion:[15]

$$\frac{d\mathbf{M}}{dt}=\gamma(\mathbf{M}\times\mathbf{H}_i)-\frac{\alpha\gamma}{|M|}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H}_i)], \tag{7}$$

where the magnetic field $\mathbf{H}$ is *internal* magnetic field, in infinite sample being equal to the external field. In the small-signal approximation the static magnetization equals the saturation magnetization. Hereafter we will adopt the usual denotation where the saturation magnetization, which defines the z axis, and the static magnetic field H_o are parallel, with the excitation rf field $\mathbf{h}$ in the perpendicular direction (x,y).

Solving of Laplace equation and boundary conditions on the ellipsoid gives the internal magnetic field in terms of external magnetic field, magnetization, and the demagnetization factors. Hence, in derivation of the ferromagnetic susceptibility of ellipsoid Kittel added demagnetization fields to both static and rf components of the external magnetic field:[15,20]

$$H_i=H_o-N_zM_s,$$
$$\mathbf{h}_i=\mathbf{h}_{\text{ext}}-\mathbf{N}\cdot\mathbf{m}. \tag{8}$$

Separating the static and rf components of magnetization and magnetic field,

$$\mathbf{M}=M_S\mathbf{e}_z+(m_x\mathbf{e}_x+m_y\mathbf{e}_y)e^{i\omega t},$$
$$\mathbf{H}_{\text{ext}}=H_o\mathbf{e}_z+(h_x\mathbf{e}_x+h_y\mathbf{e}_y)e^{i\omega t}, \tag{9}$$

where ω is a frequency of the rf magnetic field, and combining with Eqs. (7) and (8) gives the relation between the transversal $(x,\ y)$ components of the magnetization $\mathbf{m}$ and the external rf magnetic field $\mathbf{h}$. This relation can be conveniently written by introducing the susceptibility tensor

$$\mathbf{m}=\boldsymbol{\chi}\cdot\mathbf{h}_{\text{ext}}, \tag{10}$$

where tensor and its components are given by Eqs. (2) and (3). Here it is essential that the susceptibility tensor $\boldsymbol{\chi}$ is defined with a relation between the rf magnetization and the *external* rf magnetic field.[15,16]

By noting that rf magnetic dipole moment is defined as $\boldsymbol{\mu}_p=\mathbf{m}V_p$, multiplying Eq. (7) with V_p on both sides and again going through derivation of Eq. (10), it is straightforward to see that similar equation connects also the rf magnetic dipole moment of a particle and the external rf magnetic field:

$$\boldsymbol{\mu}_p=\boldsymbol{\chi}V_p\mathbf{h}_{\text{ext}}. \tag{11}$$

where V_p is a particle volume and $\boldsymbol{\chi}$ is the particle susceptibility given by Eqs. (2) and (3) with appropriate demagnetization factors.

To determine the effective susceptibility of a composite from the above equations, we follow Maxwell's derivation[1,2] and again consider a sphere of dilute composite with n spherical particles, which are made of identical ferromagnetic material and sufficiently separated to neglect interactions. The rf magnetic field or, equivalently, rf magnetic potential outside the composite sample is obtained by summing the dipole fields (potential) of every particle with rf magnetic dipole moment $\boldsymbol{\mu}_p=\mathbf{m}V_p$. However, at sufficiently large distance this dipole field is equal to the dipole field of a homogeneous spherical body with an effective rf magnetic dipole moment $\boldsymbol{\mu}_{\text{eff}}=\Sigma\boldsymbol{\mu}_p$. Thus, by writing an equation equivalent to Eq. (11) for the effective magnetic dipole of the composite sphere, one can define the effective-susceptibility tensor of the composite $\boldsymbol{\chi}_{\text{eff}}$:

$$\boldsymbol{\mu}_{\text{eff}}=\boldsymbol{\chi}_{\text{eff}}V_{\text{sample}}\mathbf{h}_{\text{ext}}$$
$$=\sum\boldsymbol{\mu}_p=\boldsymbol{\chi}\left(\sum V_p\right)\mathbf{h}_{\text{ext}}. \tag{12}$$

From Eq. (12) we can obtain an explicit expression for the effective-susceptibility tensor of the composite $\boldsymbol{\chi}_{\text{eff}}$ as a linear function of particle susceptibility tensor $\boldsymbol{\chi}$:

$$\boldsymbol{\chi}_{\text{eff}}=\left(\sum V_p\right)\boldsymbol{\chi}/V_{\text{sample}}=F\boldsymbol{\chi}. \tag{13}$$

From this equation it is evident that the resonance frequency of the composite is identical to that of the individual particle, without any frequency shift. This is in line with arguments presented in the preceding section.

In special case of composite with single-domain particles but without strong static external magnetic field the static magnetization and hence the local z axis can be arbitrarily oriented. Consequently, the effective dipole moment of the sample sphere is a sum of particles' dipole vectors and the effective susceptibility is obtained by averaging over orientation distribution:

$$\boldsymbol{\chi}_{\text{eff}}=\frac{V_p}{V_{\text{sample}}}\int\boldsymbol{\chi}(\theta,\varphi)f(\theta,\varphi)d\theta d(\sin\varphi). \tag{14}$$

Although significantly more complex, the above principles should apply also for the case of the demagnetized or partially magnetized particles.

IV. MAGNETOSTATIC DERIVATION OF EFFECTIVE SUSCEPTIBILITY

The derivation of Maxwell-Garnett equation is based on magnetostatic calculations for a spherical (ferro)magnetic particle in an external magnetic field, i.e., on solving the Laplace equation for the magnetic potential.[1,2,21,22] The magnetic potential outside the particle is calculated as a function of the permeability of the particle and the composite's potential is again obtained by a summation of the single-particle potentials. Similarly, a spherical composite sample can be viewed as an effective magnetic sphere with an effective permeability and it's magnetic potential is given as a function of this effective permeability. Equating this potential with the sum of single-particle potentials gives the Maxwell-Garnett equation [Eq. (1)].

This approach neglects one crucial characteristic of the ferromagnetic material. Unlike dielectric material, in the ferromagnetic material Eq. (7) nonlinearly couples the components of the magnetization and the external magnetic field, both static and dynamic. So, the tensor form of susceptibility (permeability) should be used in magnetostatic calculations, as suggested also by other authors.[23–25] In addition, the tensor components depend on the static (z-axis) magnetic field and so the magnetostatic calculation for z axis should also be taken into account.

However, one should be aware of an additional aspect. In magnetostatic calculation the magnetic-flux density $\mathbf{b}_i$ inside the ellipsoid is expressed with the *local* magnetic field $\mathbf{h}_i$ inside the particle[26]

$$\mathbf{b}_i=\mu_o(\mathbf{h}_i+\mathbf{m})=\mu_o(\mathbf{h}_i+\boldsymbol{\chi}_l\cdot\mathbf{h}_i),\qquad \boldsymbol{\chi}_l=\begin{vmatrix}\chi_l & -\kappa_l\\ \kappa_l & \chi_l\end{vmatrix}, \tag{15}$$

where we introduce $\boldsymbol{\chi}_l$ as local susceptibility tensor connecting rf magnetization and local rf magnetic field:

$$\mathbf{m}=\boldsymbol{\chi}_l\cdot\mathbf{h}_i. \tag{16}$$

This local susceptibility is *not* equal to the susceptibility from Eq. (10), given by Eqs. (2) and (3), since the susceptibility from Eq. (10) connects the rf magnetization and the external rf magnetic field.

Application of the local susceptibility tensor [Eq. (15)] in the magnetostatic equations gives after lengthy but straightforward calculation the following expression for the rf magnetic potential outside the particle:[22]

$$\phi=\left[-\mathbf{Id}+\mathbf{G}\frac{R^3}{r^3}\right]\mathbf{h}_{ext}\cdot\mathbf{r}, \tag{17}$$

$$\mathbf{G}=\frac{1}{(\chi_l+3)^2+\kappa_l^2}\begin{vmatrix}(\chi_l+3)\chi_l+\kappa_l^2 & -3\kappa_l\\ 3\kappa_l & (\chi_l+3)\chi_l+\kappa_l^2\end{vmatrix}, \tag{18}$$

where $\mathbf{Id}$ is identity matrix, r distance to the observation point, and R radius of the particle. The first part on right side of Eq. (17) is the potential of the external field and the second part is the potential due to the magnetic dipole of the particle. Far from the ferromagnetic resonance off-diagonal element κ_l vanishes and Eq. (17) transforms into equation for a potential of the particle with scalar susceptibility, identical to the one in the derivation of the original Maxwell-Garnett equation,

$$\phi=-\mathbf{h}_{ext}\cdot\mathbf{r}+\frac{\chi_l}{\chi_l+3}R^3\mathbf{h}_{ext}\cdot\frac{\mathbf{r}}{r^3}. \tag{19}$$

As before, the procedure is repeated for the effective sphere (of composite) having larger radius R_{eff} and effective susceptibility tensor $\boldsymbol{\chi}_{eff}$, thus giving the expression for potential of the effective sphere:

$$\phi_{eff}=\left[-\mathbf{Id}+\mathbf{G}_{eff}\frac{R_{eff}^3}{r^3}\right]\mathbf{h}_{ext}\cdot\mathbf{r}, \tag{20}$$

$$\mathbf{G}_{eff}=\frac{1}{(\chi_{eff}^{loc}+3)^2+(\kappa_{eff}^{loc})^2}\times\begin{vmatrix}(\chi_{eff}^{loc}+3)\chi_{eff}^{loc}+(\kappa_{eff}^{loc})^2 & -3\kappa_{eff}^{loc}\\ 3\kappa_{eff}^{loc} & (\chi_{eff}^{loc}+3)\chi_{eff}^{loc}+(\kappa_{eff}^{loc})^2\end{vmatrix}. \tag{21}$$

For sufficiently large distance from the effective sphere the dipole contribution to the effective potential is equal to the sum of dipole potentials of N equal spheres,

$$\left[\mathbf{G}_{eff}\frac{R_{eff}^3}{r^3}\right]\mathbf{h}_{ext}\cdot\mathbf{r}=N\left[\mathbf{G}\frac{R^3}{r^3}\right]\mathbf{h}_{ext}\cdot\mathbf{r} \tag{22}$$

From Eq. (22) a tensor variant of the MG equation is obtained, combining local effective-susceptibility tensor $\boldsymbol{\chi}_{eff}^{loc}$ and local susceptibility tensor $\boldsymbol{\chi}_l$,

$$\mathbf{G}_{eff}(\chi_{eff}^{loc},\kappa_{eff}^{loc})=F\mathbf{G}(\chi_l,\kappa_l), \tag{23}$$

where $\mathbf{G}_{eff}$ has the same form as $\mathbf{G}$ from Eq. (18) and volume fraction of particles is given by $F=NR^3/R_{eff}^3$. Expressing the effective-susceptibility tensor components as a function of local susceptibility from Eq. (23) is not trivial. Only at frequencies far from resonance ($\kappa_l\to 0$) this expression simplifies to the MG equation

$$\frac{\chi_{eff}^{loc}}{\chi_{eff}^{loc}+3}=F\left.\frac{\chi_l}{\chi_l+3}\right|_{\kappa_l\to 0}. \tag{24}$$

However, one can relate the unknown coefficients χ_l and κ_l with the susceptibility components χ and κ from Eq. (10). By writing Eq. (16),

$$\mathbf{m}=\boldsymbol{\chi}_l\cdot\mathbf{h}_l=\boldsymbol{\chi}\cdot\mathbf{h}_{ext}$$

and substituting internal (local) field with external field through Eq. (8) the following expression is obtained:

$$(\mathbf{Id}+\boldsymbol{\chi}_l\cdot\mathbf{N})\cdot\mathbf{m}=\boldsymbol{\chi}_l\cdot\mathbf{h}_{ext}.$$

By using Eq. (10) to substitute magnetization $\mathbf{m}$ with $\boldsymbol{\chi}\cdot\mathbf{h}_{ext}$ we expressed the susceptibility tensor $\boldsymbol{\chi}$ with components of the local susceptibility tensor $\boldsymbol{\chi}_l$:

$$\boldsymbol{\chi}=\frac{3}{(\chi_l+3)^2+\kappa_l^2}\begin{vmatrix}(\chi_l+3)\chi_l+\kappa_l^2 & -3\kappa_l\\ 3\kappa_l & (\chi_l+3)\chi_l+\kappa_l^2\end{vmatrix}=3\mathbf{G}. \tag{25}$$

Analogous expressions can be written also for rf magnetization of the effective sphere,

$$\mathbf{m}_{eff}=\boldsymbol{\chi}_{eff}^{loc}\cdot\mathbf{h}=\boldsymbol{\chi}_{eff}\cdot\mathbf{h}_{ext},$$

from which we obtained the expression for the effective susceptibility tensor $\boldsymbol{\chi}_{eff}$ as a function of local effective-susceptibility tensor $\boldsymbol{\chi}_{eff}^{loc}$,

$$\boldsymbol{\chi}_{\text{eff}}=\frac{3}{(\chi_{\text{eff}}^{\text{loc}}+3)^2+(\kappa_{\text{eff}}^{\text{loc}})^2}\times\begin{vmatrix}(\chi_{\text{eff}}^{\text{loc}}+3)\chi_{\text{eff}}^{\text{loc}}+(\kappa_{\text{eff}}^{\text{loc}})^2 & -3\kappa_{\text{eff}}^{\text{loc}}\\ 3\kappa_{\text{eff}}^{\text{loc}} & (\chi_{\text{eff}}^{\text{loc}}+3)\chi_{\text{eff}}^{\text{loc}}+(\kappa_{\text{eff}}^{\text{loc}})^2\end{vmatrix}=3\mathbf{G}_{\text{eff}}. \tag{26}$$

By combining this expression with Eq. (23) and (25) one can write

$$\boldsymbol{\chi}_{\text{eff}}=3\mathbf{G}_{\text{eff}}=3(F\mathbf{G})=F\boldsymbol{\chi}. \tag{27}$$

With this we obtained an expression identical to Eq. (13), which we derived from the ferromagnetic theory. Thus, we demonstrated the equivalence of both approaches and validated our derivation of a simple explicit expression for the effective ferromagnetic susceptibility [Eq. (13)].

As before, the presented arguments are valid also for the demagnetized or partially magnetized particles, but the exact magnetostatic calculation for a multidomain particle pose a considerable task even for known domain configuration.

V. DISCUSSION AND CONCLUSION

There are two reasons why the widely used effective-medium theory, valid for dielectric materials, cannot be easily applied in case of ferromagnetic materials. First, the magnetic susceptibility has a tensor form and so the effective-medium theory has to be modified for tensor calculation. Even though the modification is not challenging by itself, Eq. (23) yields a set of two nontrivial implicit equations for the evaluation of the effective-susceptibility tensor components. This tensor-form modification limits the use of the Maxwell-Garnett equation to frequencies far-off the ferromagnetic resonance, as seen from Eqs. (23) and (24).

Second, the components of the magnetic susceptibility depend on the static magnetic field in the z direction, which is perpendicular to the excitation rf field. This introduces marked difference between local susceptibility $\boldsymbol{\chi}_l$ used in magnetostatic derivation and external susceptibility $\boldsymbol{\chi}$ used in ferromagnetic derivation. Because of this difference the susceptibility expressions from literature (e.g. Refs. 15, 20) cannot be used in Maxwell-Garnett equation. However, this is frequently done in literature [5–9,11,13,14] in combination with application of MG equation near the resonance frequency. It is this combination of errors that yields the unphysical results, presented in Fig. 1.

Although both ferromagnetic and magnetostatic derivations give the physically correct results if properly applied, there is a vast difference between the complexity of the resulting expressions. In magnetostatic equation one has to transform the susceptibility expressions found in literature to the local susceptibility and then solve tensor equation (23). This is simplified at frequencies far from the ferromagnetic resonance; however, the resonance is usually of importance in ferromagnetic materials. On the other hand, by the ferromagnetic approach we derived a simple explicit equation (13), valid in the whole frequency range and utilizing the susceptibility expressions from literature. The simplicity of our ferromagnetic derivation is most evident when analyzing composites with demagnetized or partially magnetized particles. In view of this we propose the use of Eq. (13) for calculation of the ferromagnetic effective susceptibility in dilute and homogeneous composites with ferromagnetic inclusions.

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